FBO 29.10.2017

Mathematics 'A' Series

Solutions & Explanations

By- "CLASSMATE ACADEMY"

(1) (2) $\log\left(\frac{a-b}{2}\right) = \frac{1}{2}(\log a + \log b)$

$\Rightarrow \frac{a-b}{2} = (ab)^{1/2}$

$\Rightarrow (a-b)^2 = 4ab$

$\Rightarrow a^2+b^2-2ab = 4ab$

$\Rightarrow a^2+b^2 = 6ab$

(2) (1) $C = 12,500$

$S_1 = \frac{120}{100} \times 12,500. = S_2 + Tax$

$\Rightarrow S_2 = \frac{6}{5}(12500) - Tax$

$= \frac{6}{5}(12500) - 1250 = \frac{11}{10}(12500)$

$\Rightarrow S_2 = \frac{11}{10} C \Rightarrow$ 10% profit = 1,250.

(3) (4) $9^{2n} - 4^{2n} \equiv x^{2n} - y^{2n}$

$x^{2n}-y^{2n}$ is divisible by both $x+y$ & $x-y$

$\Rightarrow 9^{2n} - 4^{2n}$ is divisible by both $9+4$ & $9-4$

i.e 13 & 5.

(4) (2) $\alpha+\beta = -\frac{5}{2}$, $\alpha\beta = \frac{k}{2}$.

$\alpha^2+\beta^2+\alpha\beta = \frac{21}{4} \Rightarrow (\alpha+\beta)^2 - \alpha\beta = \frac{21}{4}$

$\Rightarrow \frac{25}{4} - \frac{k}{2} = \frac{21}{4} \Rightarrow k = 2.$

(5) (1) $2^{115}-1 = 32^{23}-1$ and $2^{25}-1 = 32^{5}-1$

Both are divisible by $32-1=31$.

(6) (3) $f(x) = 2x^3 - 3x^2 + x + K$ gives remainder 3 when divided by $x-2$

$\Rightarrow f(2) = 3$

$\Rightarrow 16-12+2+K=3$

$\Rightarrow K=-3$.

(7) (1) $G = B+4$ and $B+G=10$

$\Rightarrow G=7$ and $B=3$

(8) (4) $3(2u+v) = 7uv$, $3(u+3v) = 11uv$.

Let $u=0 \Rightarrow v=0$

Let $u=1 \Rightarrow v=\frac{3}{2}$.

(9) (4) Only one option 1, 4, 7 are in A.P.

And also 1, 4, 7 satisfies $f(x)=0$.

(10) (3) $x\left(\sqrt{3a+2b}\right) - x\left(\sqrt{3a-2b}\right) = \sqrt{3a+2b} + \sqrt{3a-2b}$

$\Rightarrow (x-1)\sqrt{3a+2b} = (x+1)\sqrt{3a-2b}$.

$\Rightarrow (x-1)^2(3a+2b) = (x+1)^2(3a-2b)$

$\Rightarrow (x^2+1-2x)(3a+2b) = (x^2+1+2x)(3a-2b)$

$\Rightarrow bx^2 - 3ax + b = 0$.

(11) (2)

	S	D	T
B+S	x	12	t_1
B-S		12	t_2

$t_1 + t_2 = 3$

$\frac{12}{B+S} + \frac{12}{B-S} = 3$

$\Rightarrow B+S=12$, $B-S=6$

$\Rightarrow S = 3$ km/hr

(12) (2) Inverted graph $\Rightarrow a < 0$

$\frac{-b}{2a} < 0 \Rightarrow \frac{b}{2a} > 0 \Rightarrow b < 0$

$\frac{-c}{4a} > 0 \Rightarrow \frac{c}{4a} < 0 \Rightarrow c > 0.$

$\Rightarrow a < 0, b < 0$ & $c > 0.$

(13) (4) $n \times n = 2401$

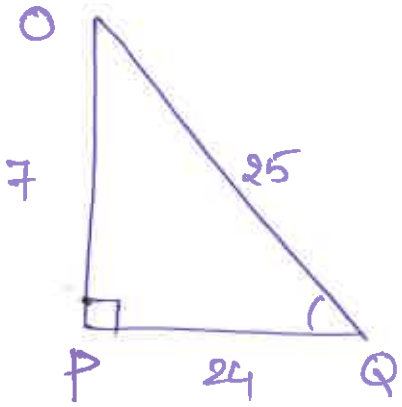
$\Rightarrow n^2 = 49^2 \Rightarrow n = 49$

(14) (1) $\sin(90^\circ+\theta) - \sin(50^\circ+\theta) + \tan 1^\circ \cdot \tan 10^\circ \cdot \tan 20^\circ \cdot \cot 20^\circ \cdot \cot 10^\circ \cdot \cot 1^\circ.$

$= 0 + 1 = 1$

(15) (2) $R = \frac{a}{\sqrt{3}} = 6 \Rightarrow a = 6\sqrt{3}.$ [R - Circum radius]

(16) (1)



$OQ - PQ = 1$

$\Rightarrow OQ = 25, \quad PQ = 24$

$\therefore \cos Q = \frac{24}{25}.$

(17) (3) $\frac{1}{\cos A}(1 - \sin A)\left(\frac{1+\sin A}{\cos A}\right) = \frac{1-\sin^2 A}{\cos^2 A} = 1$

(18) (3) $y^2 + 10^2 = 20^2 \Rightarrow y = \sqrt{400-100} = 10 \times \sqrt{3}$

$= 17.32.$

(19) (4) $\cos 1^\circ \cdot \cos 2^\circ \cdots \cos 90^\circ \cdots \cos 180^\circ = 0$

$[\cos 90^\circ = 0]$

(20) (2) $\tan 53^\circ = 1.3270$.

$\tan 53^\circ 38' > \tan 53^\circ$

$\Rightarrow \tan 53^\circ 38' = 1.3580$.

(21) (1) $\cos\theta = 0.4457$

$\cos 63^\circ = 0.4540$.

$\cos\theta < \cos 63^\circ$

$\Rightarrow \theta > 63^\circ \Rightarrow \theta = 63^\circ 32'$

(22) (2) $\sqrt{2+\sqrt{2+\sqrt{2+2\cos 8\theta}}} = \sqrt{2+\sqrt{2+2\cos 4\theta}}$

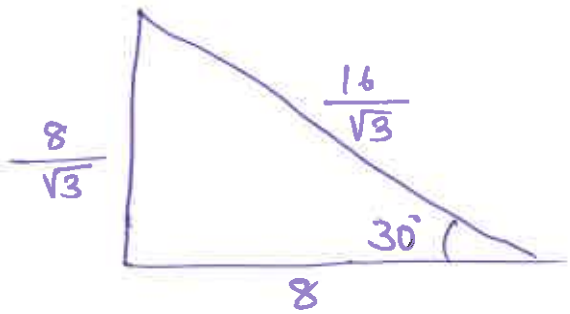
$= \sqrt{2+2\cos 2\theta} = 2\cos\theta$.

(23) (4) $m^2 - n^2 = 4\tan A \sin A$; $mn = \tan^2 A - \sin^2 A$

$= \frac{4\sin^2 A}{\cos A}$. $= \sin^2 A \cdot \left(\frac{1-\cos^2 A}{\cos^2 A}\right)$

$= 4\sqrt{mn}$ $= \frac{\sin^4 A}{\cos^2 A}$.

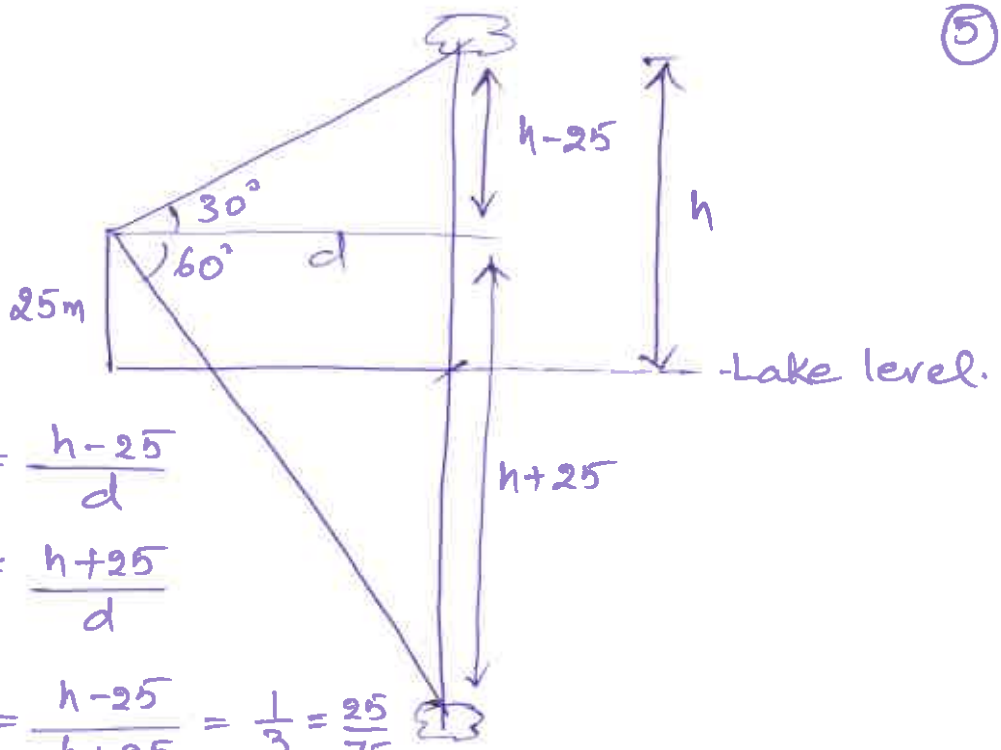
(24) (1)



$h = \frac{8}{\sqrt{3}} + \frac{16}{\sqrt{3}}$

$= \frac{24}{\sqrt{3}} = 8\sqrt{3}$ m.

(25) (4)



$\tan 30^\circ = \frac{h-25}{d}$

$\tan 60^\circ = \frac{h+25}{d}$

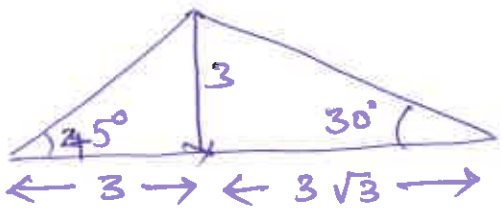
$\Rightarrow \frac{1/\sqrt{3}}{\sqrt{3}} = \frac{h-25}{h+25} = \frac{1}{3} = \frac{25}{75}$

$\Rightarrow h = 50.$

(26) (2)

$w = 3 + 3\sqrt{3}$

$= 3(\sqrt{3}+1)$ m.



(27) (3)

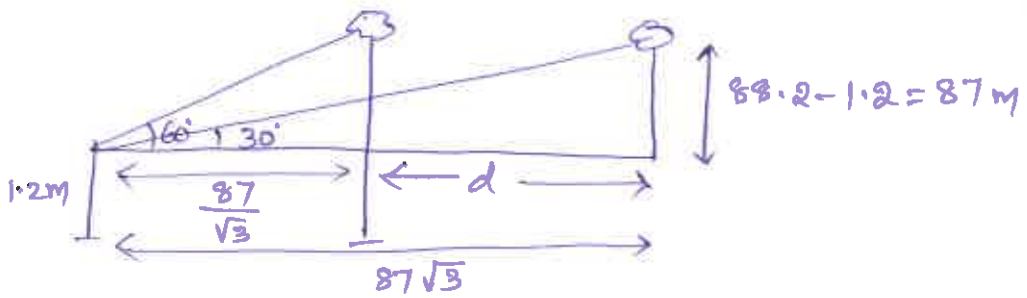
$2 \times \left(\frac{\cos 58^\circ}{\cos 58^\circ}\right) - \sqrt{3}\left(\frac{\cos 38^\circ \sec 38^\circ}{\cot 75^\circ \cdot \sqrt{3} \cdot \tan 75^\circ}\right)$

$= 2 - 1 = 1$

(28) (4)

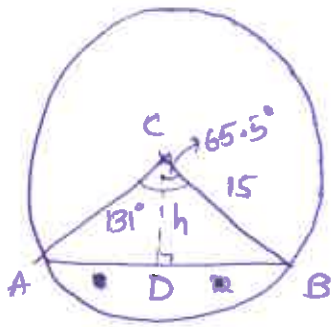
$\sqrt{\frac{\tan A \cot A + \tan A \tan A}{\sin A \cdot \operatorname{cosec} A}} = \sqrt{1+\tan^2 A} = \sec A.$

(29) (2)



$d = 87\sqrt{3} - \frac{87}{\sqrt{3}} = \frac{29(3)}{\sqrt{3}}(2) = 58\sqrt{3}$ m.

(30) (1)



$\cos 65.5^\circ = \frac{h}{15}$

$\Rightarrow h = 15 \cos 65.5^\circ$

$\cos 30^\circ = \frac{\sqrt{3}}{2} = 0.86$

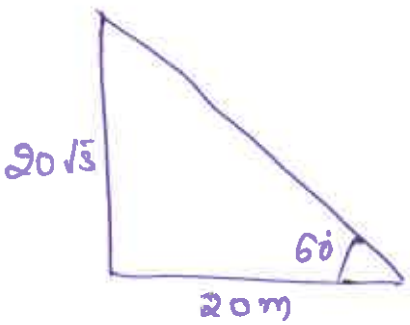
$\cos 45^\circ = \frac{1}{\sqrt{2}} = 0.7$

$\cos 60^\circ = \frac{1}{2} = 0.5$

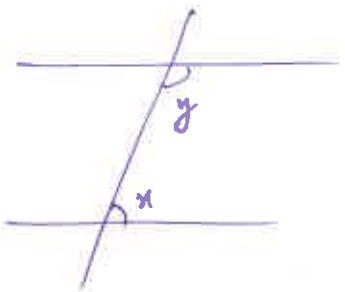
Approximation: $\cos 65.5^\circ \approx 0.42$

$\therefore h \approx 15 \times 0.42 = 6.3$.

(31) (1)



(32) (3)



$x + y = 180^\circ$

$\Rightarrow x, y$ are supplementary

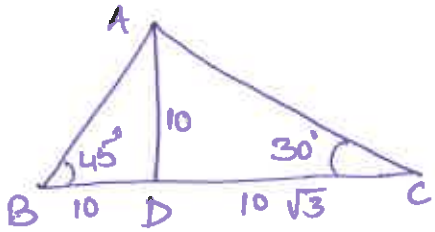
(33) (*)

$AP : PB = 1 : 2$

$P = \left(\frac{5+4}{3}, \frac{-8+2}{3}\right) = (3, -2)$

$2x - y + k = 0 \Rightarrow 6 + 2 + k = 0 \Rightarrow k = -8$

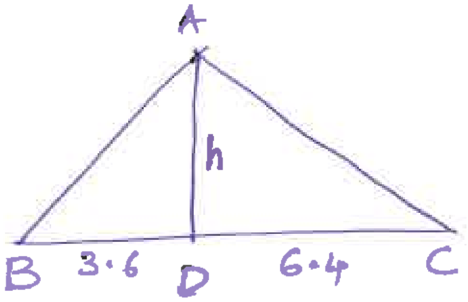
(34) (4)



$\angle A = 180^\circ - 45^\circ - 30^\circ$

$= 105^\circ$.

(35) (3)

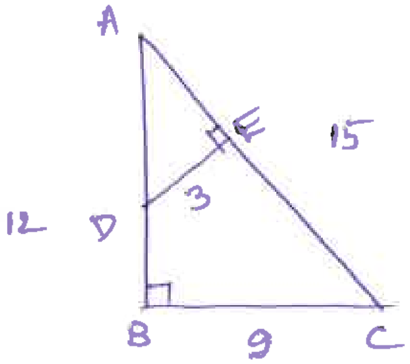


$h = \sqrt{3.6 \times 6.4}$

$= \sqrt{0.36 \times 64}$

$= 0.6 \times 8 = 4.8$ cm

(36) (2)



$\Delta ABC \sim \Delta AED$

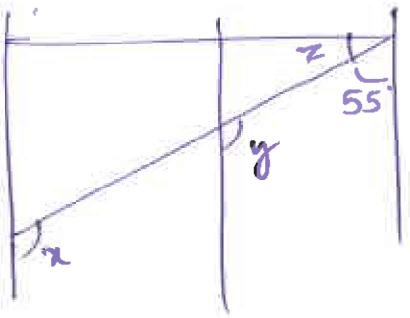
$\frac{BC}{ED} = \frac{9}{3} = \frac{3}{1}$

$\frac{\Delta_1}{\Delta_2} = \frac{3^2}{1^2} = \frac{9}{1}$

$\Delta_1 = \frac{1}{2} \times 9 \times 12 = 9 \times 6$

$\Rightarrow \Delta_2 = 6\ cm^2.$

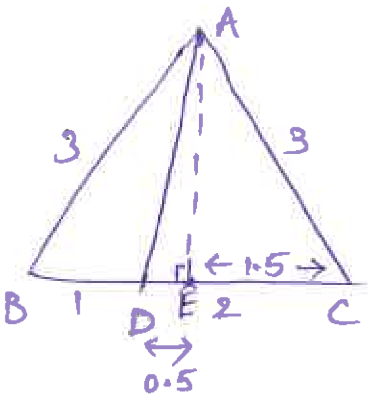
(37) (4)



$z = 90^\circ - 55^\circ = 35^\circ$

$\therefore$ option (4) is answer.

(38) (3)



$AE = \frac{\sqrt{3}}{2} \times 3$

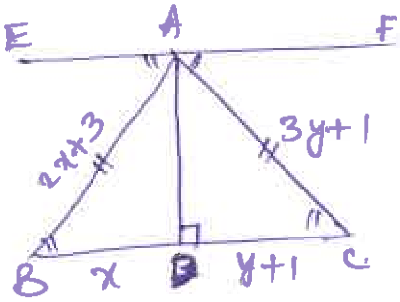
$AB^2 = 3^2 = 9$

$AD^2 = AE^2 + DE^2 = \left(\frac{\sqrt{3} \times 3}{2}\right)^2 + \left(\frac{1}{2}\right)^2$

$= \frac{27+1}{4} = 7$

$\Rightarrow \frac{AB^2}{AD^2} = \frac{9}{7} \Rightarrow AB^2 = \frac{9}{7} AD^2$

(39) (4)



$AD^2 = (2x+3)^2 - x^2 = (3y+1)^2 - (y+1)^2$

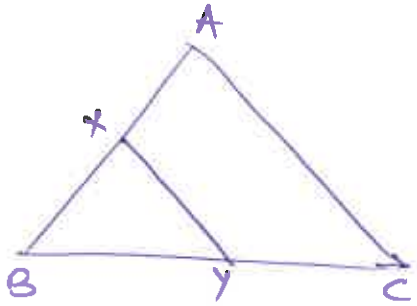
[redacted]

As $2x+3 = 3y+1 \Rightarrow x^2 = (y+1)^2 \Rightarrow x = y+1$

[redacted] $\Rightarrow 2(y+1)+3 = 3y+1 \Rightarrow y = 4$

$\Rightarrow x = 5.$

(40) (2)



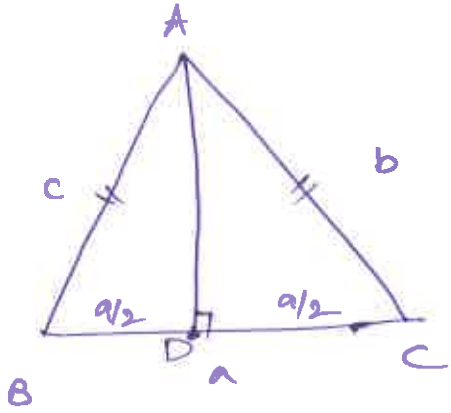
$\Delta BXY \sim \Delta BAC$

$\frac{\Delta_1}{\Delta_2} = \frac{1}{2}$ (areas)

$\Rightarrow \frac{BX}{AB} = \sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}}$

$\frac{AX}{AB} = \frac{\sqrt{2}-1}{\sqrt{2}} = \frac{2-\sqrt{2}}{2}$.

(41) * (2)



$b = c$

[If we assume equilateral then $a^2 = b^2 + c^2$

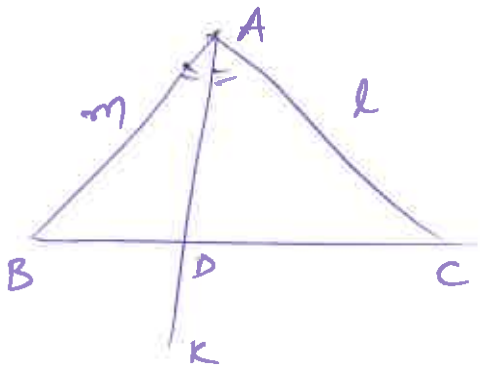
$\Rightarrow b^2 = a^2 - c^2$

$\Rightarrow b = \sqrt{a^2 - c^2}$]

[Also out of 4 options only feasible one is (2)]

(42) (4) $(x, y, z) = (1, 1, 1)$

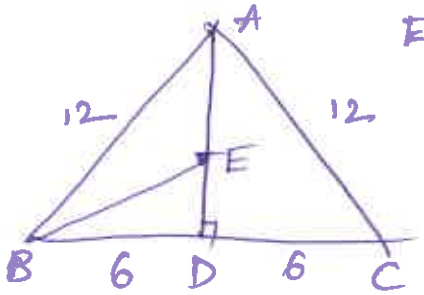
(43) (2)



$AB : AC = BD : DC = m : l$

$DC = \frac{l}{m+l}(BC) = \frac{kl}{m+l}$

(44) (1)



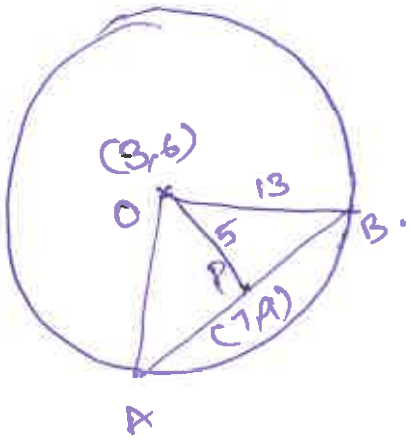
$ED = \frac{1}{2}h = \frac{1}{2} \times 12 \times \frac{\sqrt{3}}{2} = 3\sqrt{3}$

$ED = 3\sqrt{3}$

$BE^2 = ED^2 + BD^2 = 27 + 36$

$BE^2 = 63 \Rightarrow BE = \sqrt{63}$.

45 (*)



$OP = \sqrt{4^2+3^2} = 5$

$\Rightarrow PB = 12$

$AB = 24$ units

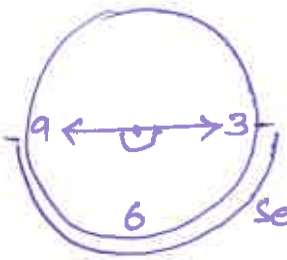
46 (1)

$AX = \sqrt{3^2+4^2} = 5$

$BX = \sqrt{3^2+4^2} = 5$

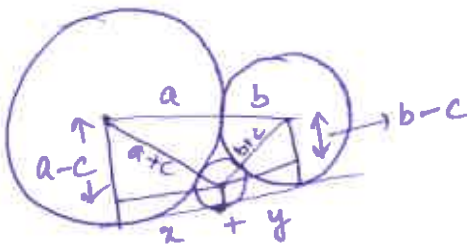
$CX = \sqrt{5^2+0^2} = 5$

47 (4)



semi circle.

48 (3)



$x+y = \sqrt{(a+c)^2-(a-c)^2} + \sqrt{(b+c)^2-(b-c)^2}$

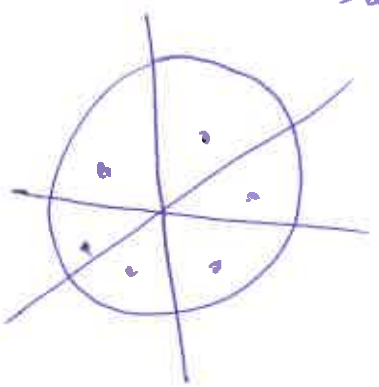
$= 2\sqrt{ac} + 2\sqrt{bc}$

$x+y$ = length of direct common tangent

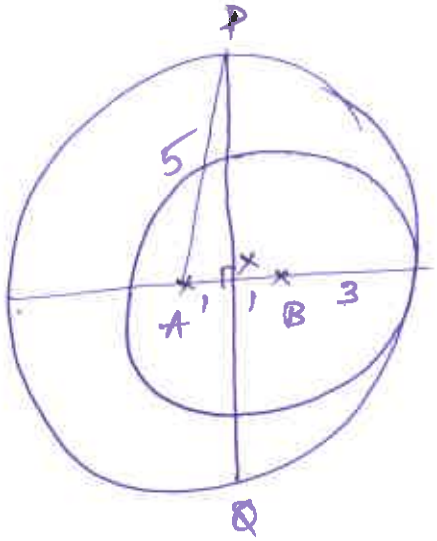
$\Rightarrow x+y = \sqrt{(a+b)^2-(a-b)^2} = 2\sqrt{ab}$

$\Rightarrow 2\sqrt{ab} = 2\sqrt{ac} + 2\sqrt{bc} \Rightarrow \frac{1}{\sqrt{c}} = \frac{1}{\sqrt{b}} + \frac{1}{\sqrt{a}}$

49 (3)



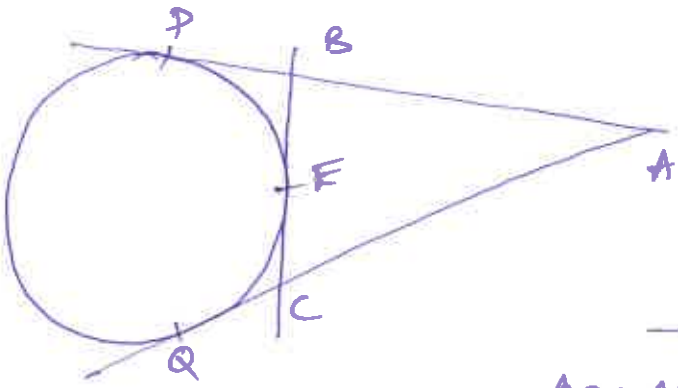
(50) (3)



$PX = \sqrt{5^2 - 1^2} = \sqrt{24} = 2\sqrt{6}$.

$\Rightarrow PQ = 2PX = 4\sqrt{6}$ cm.

(51) (1)



$AP = AQ = 5$

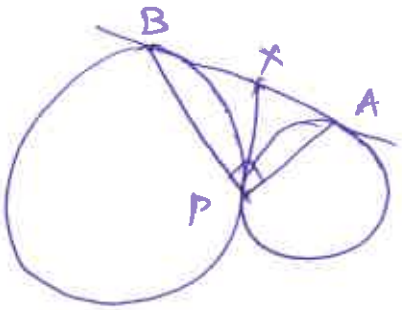
$BP = BE$ & $CE = CQ$

$\Rightarrow AQ = AC + CE$

$AP = AB + BE$

$AQ + AP = AB + BC + CA = 10$.

(52) (4)



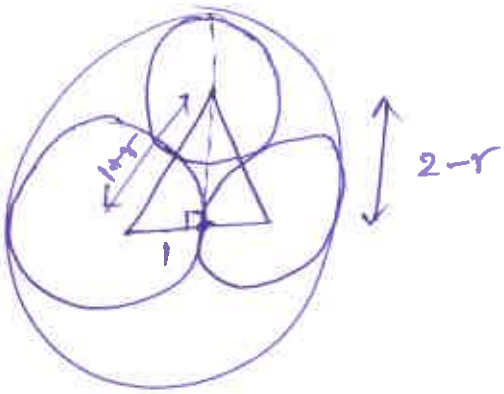
$XA = XP = XB$

$\Rightarrow$ X is mid point of AB.

$\Rightarrow \Delta APB$ is right angled Δ^{le}

$\angle P = 90^\circ$.

(53) (1)



$(1+r)^2 = (2-r)^2 + 1^2$

$\Rightarrow 1 + r^2 + 2r = 4 + r^2 - 4r + 1$

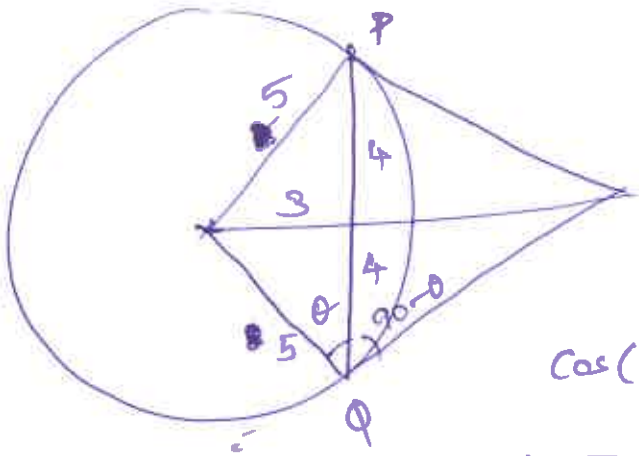
$\Rightarrow 6r = 4$

$\Rightarrow r = \frac{2}{3}$.

(54) (3)

$t = \sqrt{10^2 - (6+2)^2} = \sqrt{10^2 - 8^2} = \sqrt{36} = 6$.

(55) (2)



$TP = TQ$

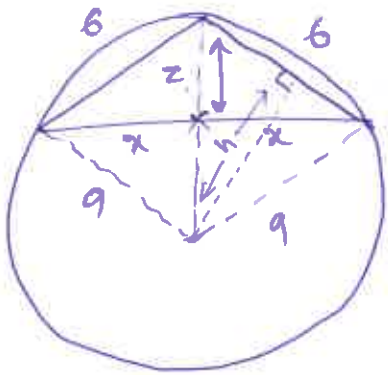
$\cos\theta = \frac{4}{5}$

$\sin\theta = \frac{3}{5}$

$\cos(90-\theta) = \frac{4}{TQ} = \frac{4}{TP}$

$\Rightarrow TP = \frac{4}{\sin\theta} = \frac{4}{3/5} = \frac{20}{3}$.

(56) (2)



$h^2 + 3^2 = 9^2 \Rightarrow h = 6\sqrt{2}$

$\frac{1}{2} \times 6 \times h = \frac{1}{2} \times 9 \times x$

$\Rightarrow \frac{2}{3}h = x \Rightarrow x = 4\sqrt{2}$

$2x = 8\sqrt{2}$

$x^2 + z^2 = 6^2 \Rightarrow z^2 = 36 - 32 = 4$

$\Rightarrow z = 2$

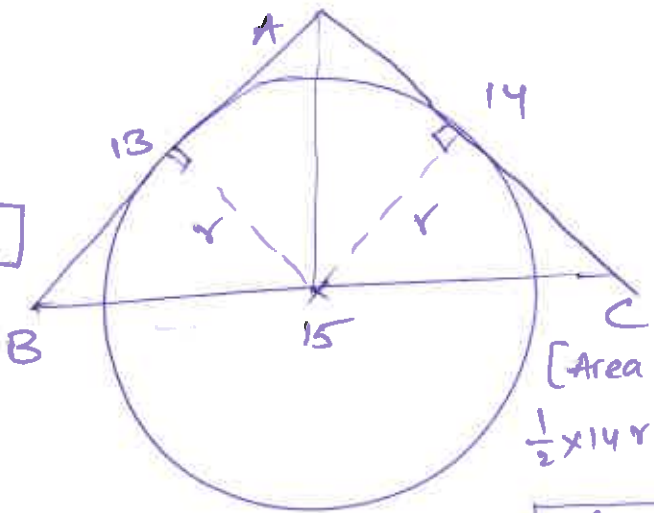
$\therefore$ Area of the triangle $= \frac{1}{2} \times (2x)\, z$

$= x \times z = 4\sqrt{2} \times 2 = 8\sqrt{2}$.

(57) (3)

(58) (*) $39\frac{1}{9}$.

[May be nearest option (4)]



[Area of ΔABC]

$\frac{1}{2} \times 14r + \frac{1}{2} \times 13r$

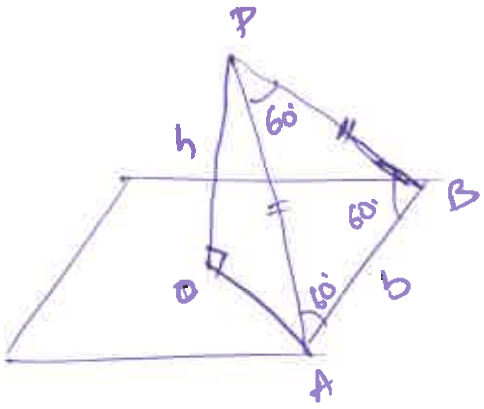
$= \sqrt{21(6)(7)(8)}$

Circumference $= 2\pi r$

$\Rightarrow r = \frac{56}{9}$

$= 2 \times \frac{22}{7} \times \frac{56}{9} = \frac{352}{9} = 39\frac{1}{9}$

(59) (2)



$PA = b$

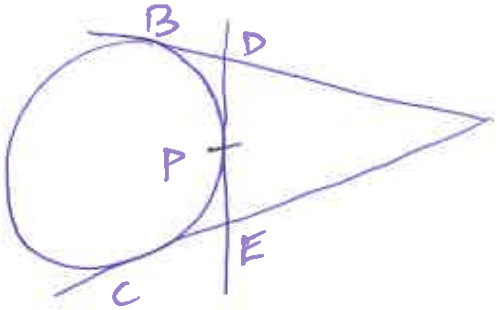
$OA = \frac{\sqrt{2}b}{2} = \frac{b}{\sqrt{2}}$

$OA^2 + h^2 = PA^2$

$\Rightarrow \frac{b^2}{2} + h^2 = b^2$

$\Rightarrow h^2 = \frac{b^2}{2} \Rightarrow 2h^2 = b^2.$

(60) (3)



$AC = AB$

$AD + BD = AE + CE$

$\Rightarrow$ ~~$AD = AE$~~

or $AB = AE + PE$

$8 = AE + 3$

$\Rightarrow AE = 5.$

(61)* (4)

$V' = \frac{4}{3}\pi(2r)^3 = 8\left(\frac{4}{3}\pi r^3\right) = 8V$

Increased to 8 times

Increased by 7 times.

[Question is not clear but in these questions mostly * increased by]

(62) (4)

$S = \pi r l \qquad S' = \pi\left(\frac{120}{100}r\right)(2l)$

$= \frac{240}{100}\pi r l = \frac{240}{100}S$

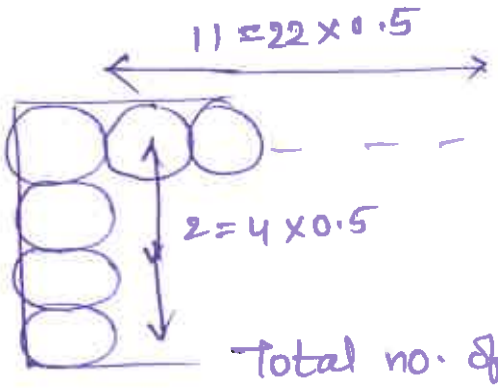
Increased by 140%.

(63) (1)

$h = a \;, \; r = \frac{a}{2}$

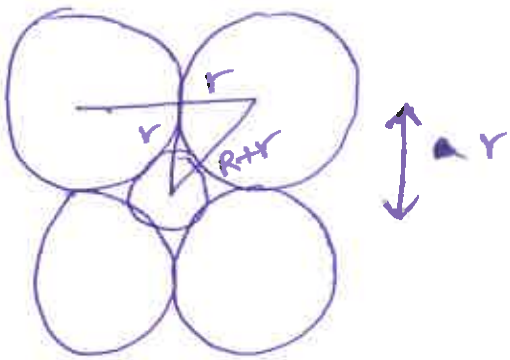
$V = \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \pi \times \frac{a^2}{4} \times a = \frac{\pi a^3}{12}.$

(64) (3)



Total no. of discs = $22 \times 4 = 88$.

(65) (2)



$(R+r)^2 = r^2 + r^2 = 2r^2$

$\Rightarrow R + r = \sqrt{2}\, r$

$\Rightarrow R = (\sqrt{2} - 1)\, r$.

(66) (3)

$2\pi R^2 = 2\pi \times \left(\frac{45.5}{2}\right)^2 = 3253.25 \text{ cm}^2$

(67) (2)

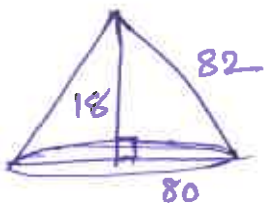
$\frac{4}{3} \times \pi \times 7^3 = 1437.33 \text{ cm}^3$.

(68) (1)

$\frac{\sqrt{3}}{4} a^2 = 3a \Rightarrow a = 4\sqrt{3} = 6.928 \simeq 6.93$.

(69) (2)

$\frac{\pi r^2}{\pi r l} = \frac{40}{41}$



$V = \frac{1}{3}\pi r^2 h = \frac{1}{3} \times 80^2 \times 18 \times \pi$

$= 38400\pi \text{ cm}^3$.

(70) (4)

$2\pi r = 11 \Rightarrow r = \frac{7}{4}$

$V = \pi r^2 h = \pi \times \frac{49}{16} \times 4 = 38.5$

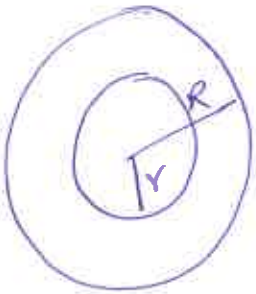
(71) (1)

$r = 4$, $h = 3$.

$V = \frac{1}{3}\pi r^2 h$

$n = \frac{V}{3.5} = \frac{\frac{1}{3} \times \frac{22}{7} \times 4^2 \times 3}{3.5} \simeq 14.$

(72) (4)



$\pi R^2 - \pi r^2 = 423.5$

$2\pi R = 132$

$r = 17.5$ cm.

(73) (3)

$\text{Ratio} = \frac{a^3}{\frac{4}{3}\pi\left(\frac{a}{2}\right)^3} = \frac{6}{\pi}$.

(74) (4)

$A = \frac{30^\circ}{360^\circ} \times \pi r^2 = 4.19 \text{ cm}^2$

(75) (2)

$x + y + \frac{xy}{100} = 10 - 12 - \frac{120}{100} = -2 - 1.2 = -3.2.$

Change is 3.2%.

(76) (3)

$\frac{4}{3}\pi (4.2)^3 = \pi 6^2 \times h$

$\Rightarrow h = 2.744$ cm.

(77) (4) $\text{mode} = l + h\left(\frac{f_m - f_1}{2f_m - f_1 - f_2}\right)$

l = lower bound of modal class

h = size of class

f_m = frequency of modal class

f_1 - preceding f_2 - succeeding

$\text{mode} = 3 + 2\left(\frac{8-7}{16-7-2}\right) = 3 + \frac{2}{7} = 3.2857.$

(78) (3) $\Sigma f = 200$ $f_1 + f_2 = 200 - (46+25+10+5)$

$= 114$

mean $= 73$

$73 = \frac{1}{200}[50f_1 + 100f_2 + 150\times25 + 200\times10 + 250\times5]$

$\Rightarrow f_1 + 2f_2 = 152$

On solving we get, $f_2 = 38$, $f_1 = 76$.

(79) (2) $\frac{\Sigma x}{100} = 40 \Rightarrow \Sigma x = 4000$

New $\Sigma x = 4000 - 83 + 53 = 3970$.

New mean $= \frac{3970}{100} = 39.7$

(80) (4) Median of original data $= \frac{36+37}{2} = 36.5$

When 35 is removed, median $= 37$

$\Rightarrow$ median is increased by 0.5

(81) (1) Mode $= 3$ median $- 2$ mean

Mode $-$ median $= 2$(median $-$ mean)

$\Rightarrow 24 = 2$ (median $-$ mean)

$\Rightarrow$ median $-$ mean $= 12$

(82) (2) Mean $-$ mode $= 3$(mean $-$ median)

$-12 = 3$(mean $-$ median)

$\Rightarrow$ mean $-$ median $= -4$

$\Rightarrow$ Mean $-$ mode $=$ Median $-$ mode $- 4$

$\Rightarrow$ Mode $-$ Median $= 12 - 4 = 8$.

(83) (1) No. of students = 40

$a+b = 40-(6+16+13)$

$a+b = 5$ — (1)

$\text{Mean} = \frac{1}{40}[5(6)+6a+7(16)+8(13)+9b]$

$6a+9b = 42$ — (2)

On solving (1) & (2) we get

$a=1, b=4.$

(84) (3) Last digits of the given terms is $1+2+3+\cdots+99$

$\therefore$ last digit of $1^5+2^5+\cdots+99^5$

= last digit of $1+2+\cdots+99$

= last digit of $\frac{99\times100}{2} = 99\times50$

$= 0.$

(85) (4) Let $n = 198$

$2^{200} - 2^{192}\cdot 31 + 2^{198}$

$= 2^{192}\left(2^8 - 31 + 2^6\right) = 2^{192}\times 289$

which is a perfect square.

(86) (3) $a^2-b^2 = (a+b)(a-b)$

~~a+b is even and a-b is also even~~

$\therefore (a+b)(a-b)$ is a composite number.

87 (2) LCM + HCF = 1260.

LCM − HCF = 900.

2 LCM = 2160 ⇒ LCM = 1080.

HCF = 180

$a \times b$ = LCM × HCF = 1080 × 180 = 194400.

88 (2) $4x^2 + 7xy - 2y^2 = 4x^2 - xy + 8xy - 2y^2$

$= x(4x - y) + 2y(4x - y)$

$= (4x - y)(x + 2y)$

which is always divisible by '3'.

89 (1) 114345 is divisible by both 9 & 11 i.e 99.

90 (3) $\frac{N}{D} = \frac{3}{7} = \frac{9}{21}$ $\quad \frac{N+6}{D+6} = \frac{5}{9} = \frac{15}{27}$

91 (1) $9S = 11C \Rightarrow \frac{S}{C} = \frac{11}{9}$

Profit is $\frac{2}{9} = \frac{2}{9} \times 100\% = 22\frac{2}{9}\%$

92 (1) $5(4) + 20(-2) = 20 - 40 = -20$ marks.

93 (1) $n = 10800 = 2^4 \times 3^3 \times 5^2$ $\quad 4m+2 = 2(2m+1)$

No. of $2m+1$ form of divisors formed from $2^4 \times 3^3 \times 5^2 = (3+1)(2+1) = 12$

(94) (3) $Q \cup R = \{2,3,4,8,9,12\}$

$(Q \cup R)' = \xi - (Q \cup R) = \{1,5,6,7,10,11\}$

$(Q \cup R)' \cap P = \{5,7,11\}$.

(95) (2)

(96) (3) $\frac{n(n+1)}{2} = \frac{100 \times 101}{2} = 5050$.

(97) (4) HCF of $245-5$ and $1029-5$

$=$ HCF of 240 and 1024

$= 16$

(98) (2) Lcm of $80, 85, 90 = 12240$ cm

$= 122$ m 40 cm.

(99) (4) Lcm and HCF are equal for equal numbers only.

(100) (3) HCF of 210 and $55 = 5$

$\Rightarrow 210 \times 5 + 55y = 5$

$\Rightarrow 210 + 11y = 1$

$11y = -209$

$y = -19$.